Exercise 4 C, P77

(1) (a) $a\underline{i} + b\underline{j} = 2\underline{i} + 4\underline{j} + 3\underline{i} - 4\underline{j}$

$= 5i \text{ N} = \text{Resultant}$

(b) $a\underline{i} + b\underline{j} = 3\underline{i} - 5\underline{j} + 2\underline{i} - 5\underline{j} - \underline{i} + 7\underline{j}$

$= (4\underline{i} - 3\underline{j}) \text{ N} = \text{Resultant}$

(c) $a\underline{i} + b\underline{j} = 6\underline{i} + 2\underline{j} - 5\underline{i} + \underline{j} + 3\underline{i} - 3\underline{j}$

$= (4\underline{i}) \text{ N} = \text{Resultant}$

(d) $a\underline{i} + b\underline{j} = 2\underline{i} + 4\underline{j} + 3\underline{i} - 5\underline{j} + 6\underline{i} + 2\underline{j} - 7\underline{i} - 7\underline{j}$

$= (4\underline{i} - 6\underline{j}) \text{ N} = \text{Resultant}$

(2) (a) $a\underline{i} + b\underline{j} + c\underline{k} = 2\underline{i} + 3\underline{j} + 3\underline{k} + 2\underline{i} + 4\underline{j} - 8k$

$= (4\underline{i} + 7\underline{j} - 5\underline{k}) \text{ N} = \text{Resultant}$

(b) $a\underline{i} + + b\underline{j} + c\underline{k} = 7\underline{i} - 4\underline{j} + 3\underline{k} + 5\underline{i} - 2\underline{j} + 8\underline{k} + \underline{i} - \underline{k}$

$= (13\underline{i} - 6\underline{j} + 10\underline{k}) \text{ N} = \text{Resultant}$

(c) left as Exercise.

(3) $5\underline{i} - 2\underline{j} + 7\underline{i} + 4\underline{j} + a\underline{i} + b\underline{j} - 3\underline{i} + 2\underline{j} = 5\underline{i} + 5\underline{j}$

So $9\underline{i} + a\underline{i} + 4\underline{j} + b\underline{j} = 5\underline{i} + 5\underline{j}$

Compare $\underline{i}$ & $\underline{j}$ components: $\underline{i}$: $9 + a = 5 \Rightarrow a = -4$

$\underline{j}$: $4 + b = 5 \Rightarrow b = 1$

(4) $5\underline{i} + 7\underline{j} + a\underline{i} + b\underline{j} + b\underline{i} - a\underline{j} = 11\underline{i} + 5\underline{j}$

So $(5 + a + b)\underline{i} + (7 + b - a)\underline{j} = 11\underline{i} + 5\underline{j}$

Compare $\underline{i}$ & $\underline{j}$ components: $\underline{i}$: $5 + a + b = 11$

$\underline{j}$: $7 + b - a = 5$

Solve:

$$\begin{array}{r} a + b = 6 \\ + \quad b - a = -2 \\ \hline 2b = 4 \end{array} \Rightarrow b = 2$$

$\Rightarrow a = 4$

(5) $\underline{i} - 2\underline{j} + 2a\underline{k} + 2\underline{i} + \underline{j} + 4\underline{k} + b\underline{i} + 2\underline{j} = 8\underline{i} + c\underline{j} + 14\underline{k}$

So $(1 + 2 + b)\underline{i} + (-2 + 1 + 2)\underline{j} + (2a + 4)\underline{k} = 8\underline{i} + c\underline{j} + 14\underline{k}$

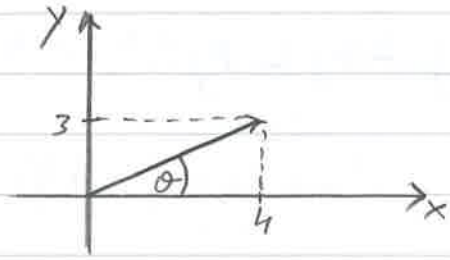
Compare $\underline{i}$ & $\underline{j}$ & $\underline{k}$ components: $\underline{i}$: $1 + 2 + b = 8$

$\underline{j}$: $-2 + 1 + 2 = c$

$\underline{k}$: $2a + 4 = 14$

So $c = 1$, $b = 5$, $a = 5$.

(6)

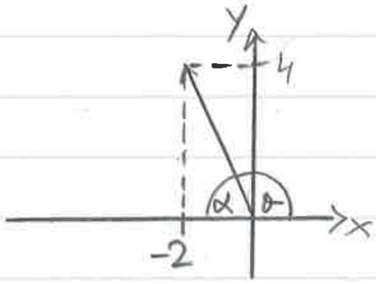


for $4\underline{i} + 3\underline{j}$

$$R = \sqrt{4^2 + 3^2} = 5\ N$$

$$\theta = \tan^{-1}\frac{3}{4} = 36.87^\circ$$

(7)



For $-2\underline{i} + 4\underline{j}$

$$R = \sqrt{(-2)^2 + 4^2} = \sqrt{20}\ N$$
$$= 4.47\ N$$

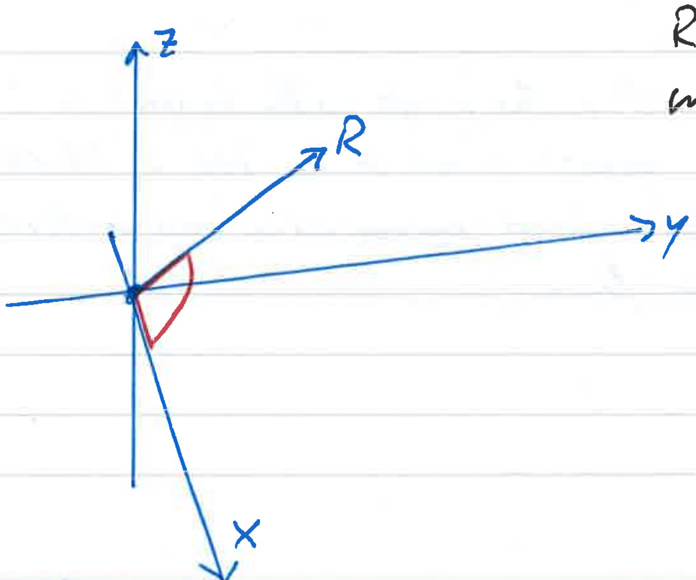
$$\alpha = \tan^{-1}\frac{4}{2} = 63.43^\circ$$

we want θ (Note that the direction of $\underline{i}$ is always from 0 to 1, i.e. on the +ve x-axis).

So $\theta = 180 - 63.43 = 116.57^\circ$

(8) given $3\underline{i} + 4\underline{j} - 5\underline{k}$, $|R| = \sqrt{3^2 + 4^2 + (-5)^2} = \sqrt{50}\ N$
$= 7.07\ N$

To find the angle this 3-D Resultant makes with the x-axis we use direction cosines. See any A-level pure maths book under "vectors"



Red angle is the angle R makes with x-axis. Then

$$\cos\theta = \frac{\underline{i}\text{ component}}{|R|}$$
$$= 3 \div 7.07$$
$$= 0.424$$
$$\Rightarrow \theta = 64.9^\circ$$

(9)

(a) $R = 2\underline{i} + 3\underline{j} + 5\underline{i} - 2\underline{j} - 3\underline{i} + 3\underline{j}$

$= (+4\underline{i} + 4\underline{j})\ N$

$|R| = \sqrt{4^2 + 4^2} = \sqrt{32} = 5.66\ N$

& $\theta = \tan^{-1} \frac{4}{4} = 45^\circ$

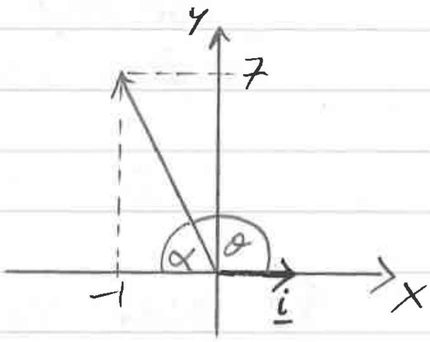
(b) $R = -2\underline{i} + 5\underline{j} + \underline{i} + 2\underline{j} = -\underline{i} + 7\underline{j}$

$|R| = \sqrt{(-1)^2 + 7^2} = \sqrt{50} = 7.07N$

& $\alpha = \tan^{-1} \frac{7}{-1} = -81.87^\circ$

So $\theta = 180 - 81.87 = 98.1^\circ$

(See diagram right for visual reason)



(c) $R = 4\underline{i} + 3\underline{j} - \underline{i} - 5\underline{j}$

$= 3\underline{i} - 2\underline{j}$, So $|R| = \sqrt{3^2 + (-2)^2} = 3.61N$

& $\theta = \tan^{-1}\left(-\frac{2}{3}\right) = -33.69^\circ$. Technically this does answer the question: θ is the angle with respect to the $\underline{i}$ direction. The book gives θ as $\theta = 360 - 33.69 = 326.3^\circ$ because the authors have measured θ as anticlockwise from $\underline{i}$ not clockwise from $\underline{i}$.

(d) $R = 2\underline{i} + 4\underline{j} - 6\underline{i} - 5\underline{j} + 2\underline{i} + \underline{j} = -2\underline{i}$

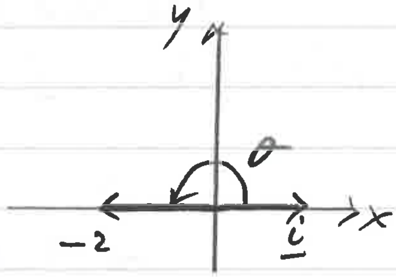
So $|R| = \sqrt{(-2)^2 + 0^2} = 2\,N$

& $\theta = \tan^{-1} \frac{0}{-2} = 0$.

But $-2i$ lies on $-ve$ x-axis & $\underline{i}$ is on $+ve$ x-axis

So

$\theta = 180^\circ$



(10) $R = 3\underline{i} + \underline{j} - \underline{k} - 4\underline{i} + \underline{j} + 4\underline{k} + 3\underline{i} + \underline{j} + 3\underline{k}$

$= +2\underline{i} + 3\underline{j} + 6\underline{k}$

$\therefore |R| = \sqrt{2^2 + 3^2 + 6^2} = 7\ N$

Now use direction cosines (see (8)). Hence

$\cos\theta = \frac{6}{7} \Rightarrow \theta = 31^\circ$

(11) (a) $R = 8\underline{i} + 10\,\underline{i}\cos 60 + 10\,\underline{j}\sin 60$

$= (13\underline{i} + 5\sqrt{3}\,\underline{j})\,N$

$\therefore |R| = \sqrt{13^2 + 25(3)} = \sqrt{244} = 15.62\ N$

& $\theta = \tan^{-1}\left(\frac{5\sqrt{3}}{13}\right) = 33.67^\circ$

(b) $R = -5\underline{i} + 5\sqrt{2}\,\underline{i}\cos 45 + 5\sqrt{2}\,\underline{j}\sin 45$

$= (+5\underline{j})\ N$

So $|R| = \sqrt{(0)^2 + 5^2} = 5\ N$

$\nearrow \quad \theta = \tan^{-1}\left(\frac{+5}{0}\right) = +90^\circ$.

(c) $R = \underline{i}\,(2\sqrt{2}\cos 45 + 6\cos 30 - 4\cos 60)$

$+ \underline{j}\,(2\sqrt{2}\sin 45 - 6\sin 30 - 4\sin 60)$

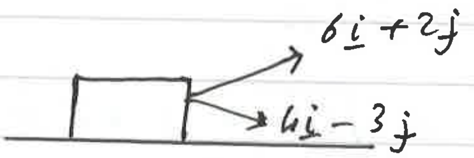
$= [3\sqrt{3}\,\underline{i} - (1+2\sqrt{3})\,\underline{j}]\ N$

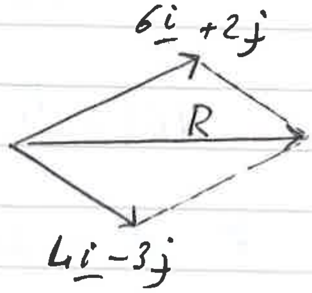
$\therefore |R| = \sqrt{(3\sqrt{3})^2 + (-1-2\sqrt{3})^2} = 6.85N$

$\nearrow \quad \theta = \tan^{-1}\left(\frac{-1-2\sqrt{3}}{3\sqrt{3}}\right) = -40.67^\circ$

Book answer $= 319.33^\circ$ because they are measuring the angle anticlockwise from +ve x-axis. But the question does not specify this.

(12)





$R = 4\underline{i} - 3\underline{j} + 6\underline{i} + 2\underline{j}$

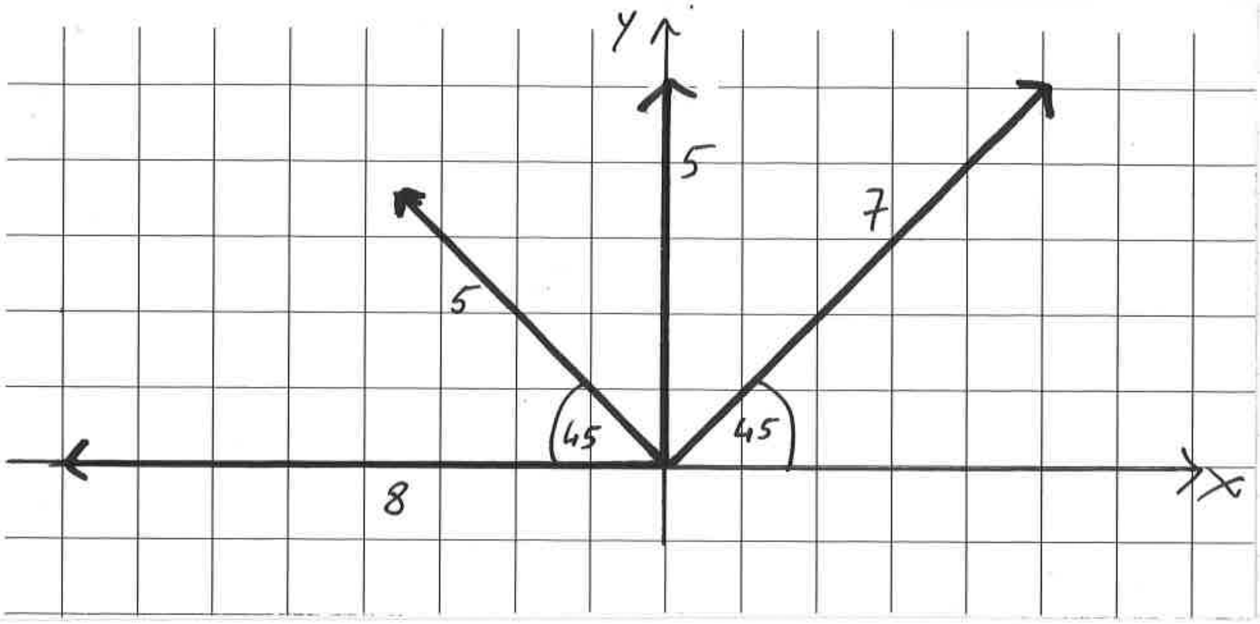
$= 10\underline{i} - \underline{j}$

$\Rightarrow |R| = \sqrt{10^2 + (-1)^2} = 10.05N$

8 $\theta = \tan^{-1} \frac{-1}{10} = -5.7°$ below x-axis

OR $\theta = 360 - 5.7 = 354.3°$ anticlockwise from +ve x-axis.

(13)



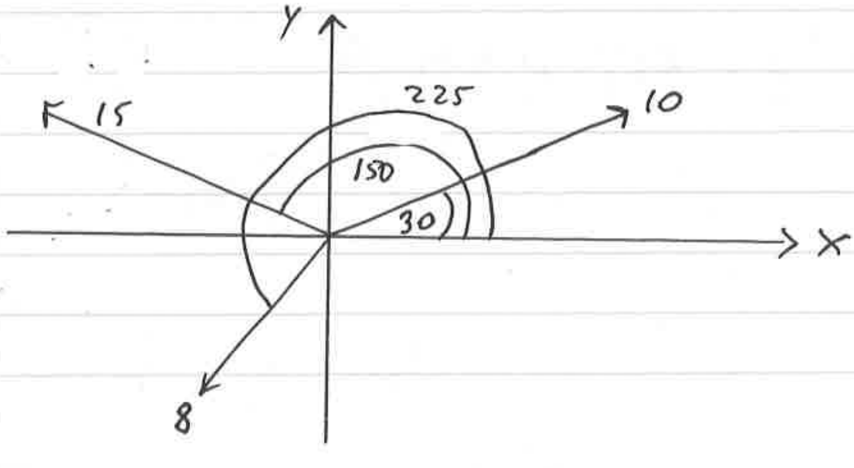
So $R = \underline{i}(-8 - 5\cos 45 + 7\cos 45)$

$+ \underline{j}(5 + 5\sin 45 + 7\sin 45)$

$= \left[(-8+\sqrt{2})\underline{i} + (5+6\sqrt{2})\underline{j}\right]$ N

$\therefore |R| = \sqrt{(-8+\sqrt{2})^2 + (5+6\sqrt{2})^2} = 16.45$ N

(14)



So $R = \underline{i}\,(10\cos 30 - 15\cos 30 - 8\cos 45)$

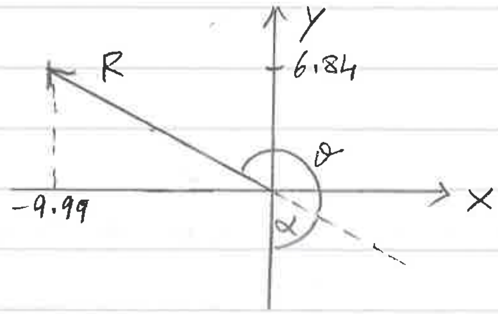
$+ \underline{j}\,(10\sin 30 + 15\sin 30 - 8\sin 45)$

$= -9.99\,\underline{i} + 6.84\,\underline{j}$

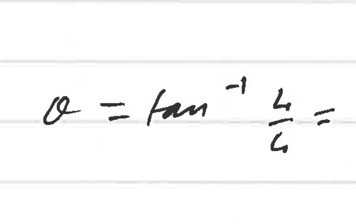
So $|R| = \sqrt{(-9.99)^2 + 6.84^2} = 12.1\,N$

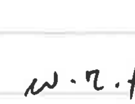
Direction w.r.t. x-axis : $= \tan^{-1}\left(\dfrac{6.84}{-9.99}\right) = -34.4^\circ$

So Required angle $\theta = 180 - 34.4 = 145.6^\circ$. See diag Below



(15)





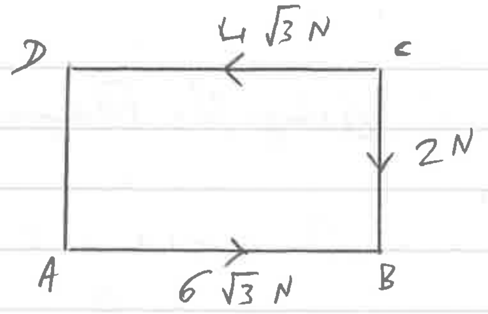
: $R = (3+1)\underline{i} + 4\underline{j}$

$= 4\underline{i} + 4\underline{j}$

$\therefore |R| = \sqrt{4^2 + 4^2} = 5.66\ N$

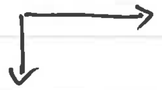
& $\theta = \tan^{-1}\frac{4}{4} = 45^\circ$ w.r.t. AB.

(16)

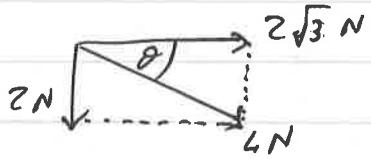


Resolve Parallel to AB : horizontal component $= 6\sqrt{3} - 4\sqrt{3}$
$= 2\sqrt{3}$ N

Resolve Parallel to CB : vertical component $=$ 2N

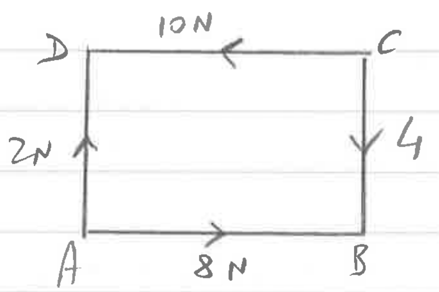
(i.e. our axes / Frame of Ref is )

So $|R| = \sqrt{(2\sqrt{3})^2 + 2^2} = 4$ N

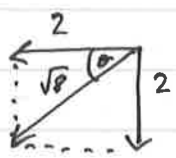
So we have R located as: 

So $\theta = \tan^{-1}\left(\frac{2}{2\sqrt{3}}\right) = 30^\circ$ w.r.t AB (under AB)

(17)



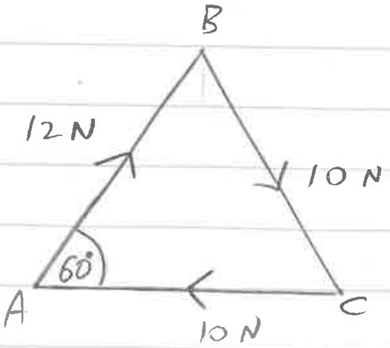
Resolve parallel to AB : horizontal component $= 8 - 10 = -2$ N
" " " AD : vertical " $= 2 - 4 = -2$ N

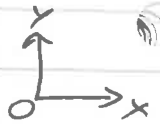
So R is located as here : 

$\Rightarrow R = \sqrt{2^2 + 2^2} = \sqrt{8}$

So $\theta = \tan^{-1}\left(\frac{2}{2}\right) = 45^\circ$ w.r.t AB to the left of A.

(18)

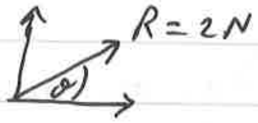


using The Frame of Ref 

$$O_x = -10 + 12\cos 60 + 10\cos 60$$
$$= 1\ N$$

$$O_y = 12\sin 60 - 10\sin 60$$
$$= \sqrt{3}\ N$$

So $|R| = \sqrt{1^2 + (\sqrt{3})^2}$

$= 2N$, So 

& $\theta = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = 60°$ i.e parallel to AB.

(19) - (20) left as Ex.